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Test 3

Calculus of Trigonometric Functions
Discrete Random Variables
Binomial Distributions
Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

Name: CHENG / Version 2

Teacher:

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_____ Mr Strain

Date: Wed 2nd May

You may have a formula sheet for this section of the test.
Classpad Calculators
1 page of Notes

Total_____/41 **45 minutes +5 minutes READING**

Question 1 **(5 marks)**

The discrete random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$P(X=x)$	$\frac{2a^2}{3}$	$\frac{1-3a}{3}$	$\frac{1+2a}{3}$	$\frac{4a^2}{3}$

Determine the value of the constant a.

Question 2 **(8 marks)**

(a) Differentiate $e^{-3x}\sin(2x)$ with respect to x, showing full working. (2 marks)

(b) Hence find the following indefinite integral. (3 marks)

$$-3\int e^{-3x}\sin(2x)\,dx + 2\int e^{-3x}\cos(2x)\,dx.$$

And using a similar process as part (a), find the indefinite integral for

$$-3\int e^{-3x}\cos(2x)\,dx - 2\int e^{-3x}\sin(2x)\,dx.$$

(c) Use the two equations from (b) to determine $\int e^{-3x}\sin(2x)\,dx$. (3 marks)

Question 3 **(6 marks)**

Differentiate with respect to x, (show full working)

(a) $y = \sin^3(2x+1)$. (3 marks)

Evaluate the following, showing full working.

(b) $\int_{\pi/6}^{\pi/2} cos(2x)\,dx$ (3 marks)

Question 4 **(9 marks)**

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable X be the number of first grade avocados in a single tray.

(a) Explain why X is a discrete random variable, and identify its probability distribution. (2 marks)

(b) Calculate the mean and standard deviation of X. (2 marks)

(c) Determine the probability that a randomly chosen tray contains

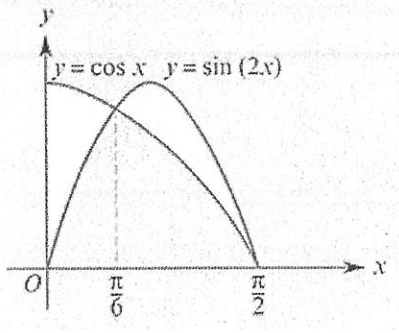
(i) 18 first grade avocados. (1 mark)

(ii) more than 15 but less than 20 first grade avocados. (2 marks)

(d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

Question 5 **(4 marks)**

Find the area between the two curves from $0 \leq x \leq \frac{\pi}{2}$, showing full algebraic reasoning.



Question 6 **(9 marks)**

(a) A sample of six objects is to be drawn from a large population in which 20% of the objects are defective. Find the probability that the sample contains:

(i) three defectives. (2 marks)

(ii) fewer than three defectives. (2 marks)

(b) Another large population contains a proportion p of defective items.

(i) Write down an expression in terms of p for P, the probability that a sample of six items contains exactly two defectives. (2 marks)

(ii) By differentiating to find $\frac{dP}{dp}$, show that P is greatest when $p = \frac{1}{3}$. (3 marks)